Efficiency improvement in proton dose calculations with an equivalent restricted stopping power formalism

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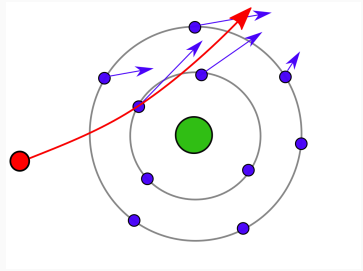
The proton energy loss along a step length

Electromagnetic inelastic interactions with atomic electrons



- Split in **soft** and **hard** collisions.

	Soft collisions	**Hard collisions**
Ionization ranges	short (sub-cutoff)	significant
Transport	Deterministic	Monte Carlo
Physical quantities	Stopping power	Production cross section
Simulation scheme	Condensed	Analog

The proton Continuous Slowing Down Approximation (CSDA)

- An approximate variance reduction technique to compute the **mean proton energy loss** (ΔE)
- The **true** energy loss (ΔE_{true}) is obtained with the energy **straggling** ($\delta_{\Delta E}$):

$$\Delta E_{true} = \Delta E \pm \delta_{\Delta E}$$

The mean energy loss (ΔE) calculations

Two schemes

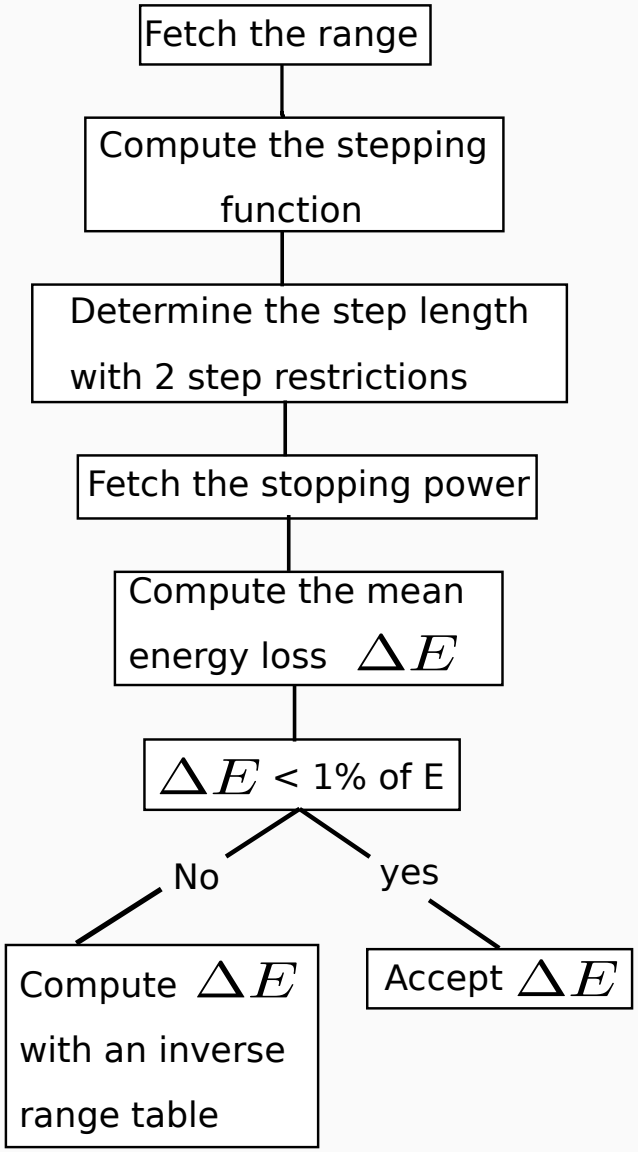
EGSnrc for electrons/positons (Kawrakow, 2000; Kawrakow et al., 2016) and **Geant4** for all charged particles (Collaboration, 2015).

A new one proposed

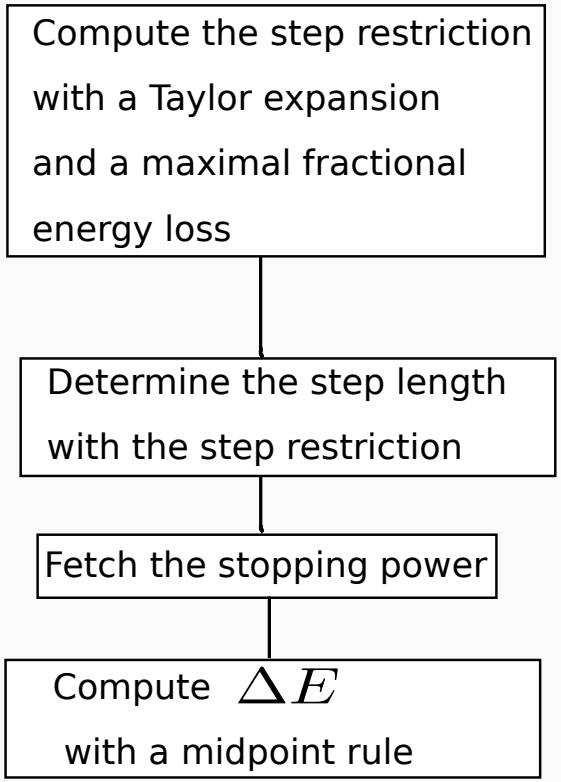
named the equivalent restricted stopping power formalism: **the L_{eq} formalism**

The ΔE calculations

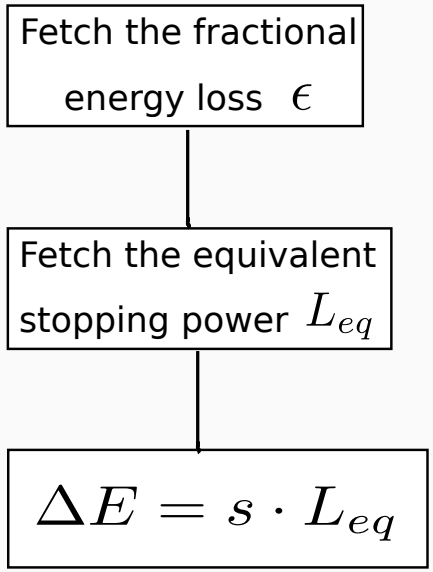
Geant4



EGSnrc



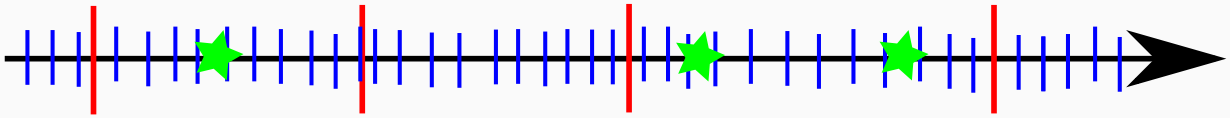
L_{eq} formalism



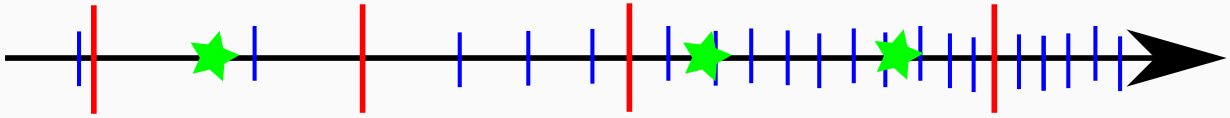
L_{eq} formalism benefit

Step length restriction | Interface | hard collision | proton trajectory

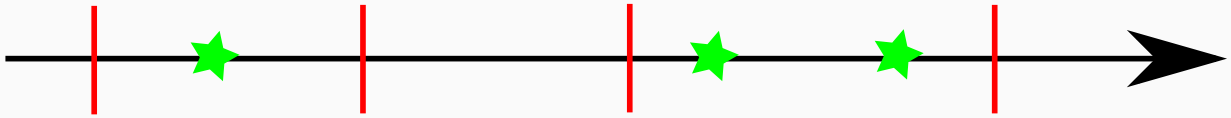
High dose accuracy simulations:



Balanced simulations: accuracy - computation time:



High accuracy with the L_{eq} formalism:



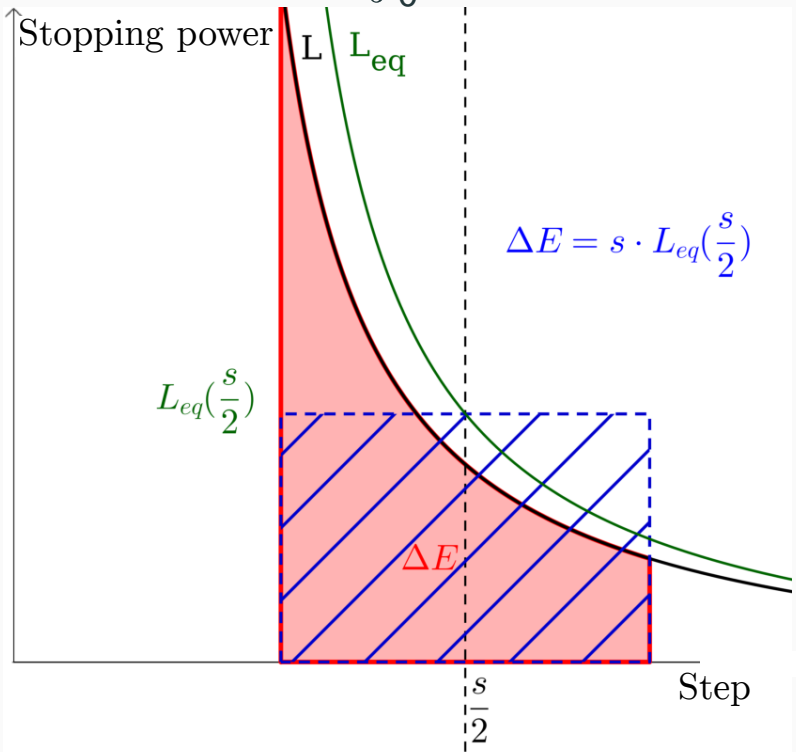
Geant4 or EGSnrc	The L_{eq} formalism
$s = min(d_{vox}, d_{hard}, d_{max})$	$s = min(d_{vox}, d_{hard})$

The L_{eq} formalism

L is the restricted/unrestricted stopping power. E is the proton kinetic energy

The distance (s) space

$$\frac{d\Delta E}{ds} = L(s)$$
$$\Leftrightarrow \Delta E = \int_0^s L(E')\,\mathrm{d}s'$$



The energy (ΔE) space

$$\frac{ds}{d\Delta E} = L^{-1}(E - \Delta E)$$
$$\Leftrightarrow s = -\int_E^{E-\Delta E} L^{-1}(E')\mathrm{d}E'$$

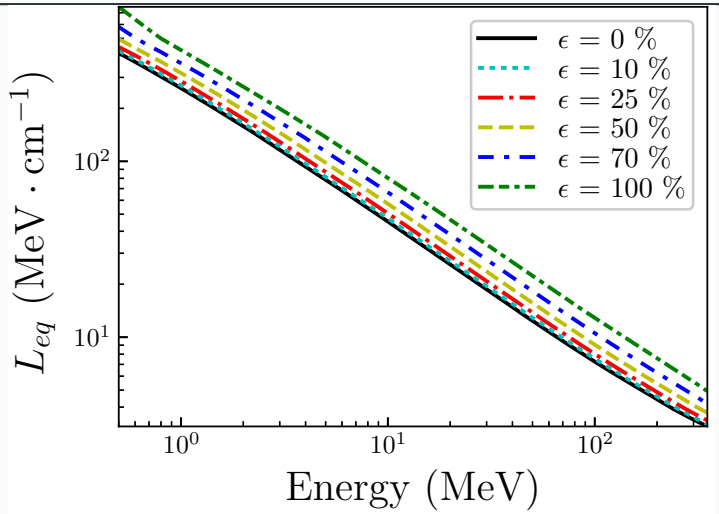
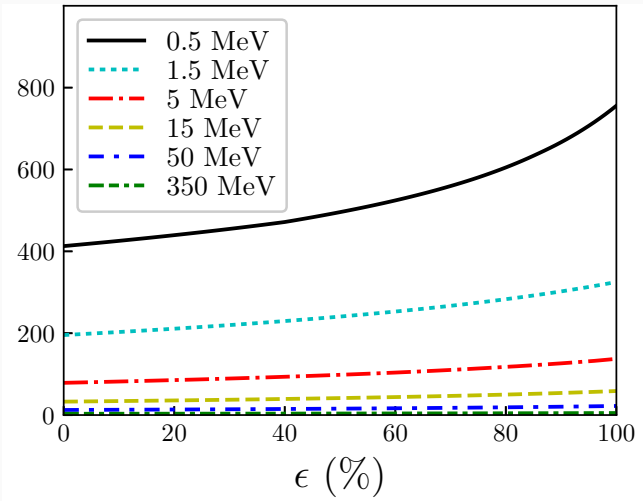
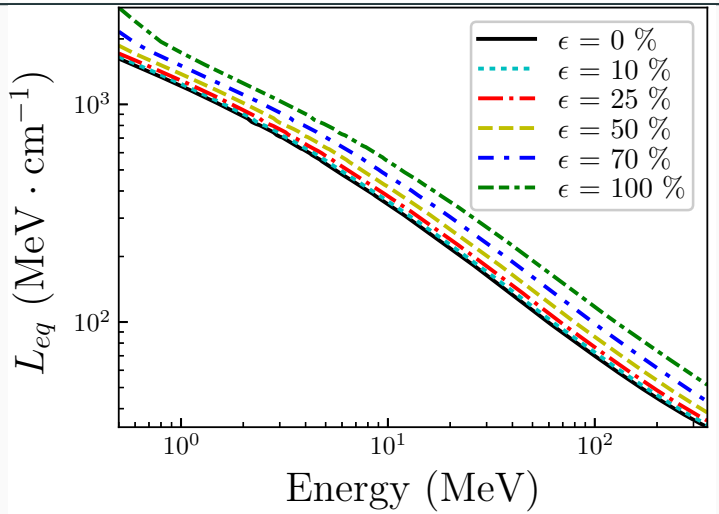
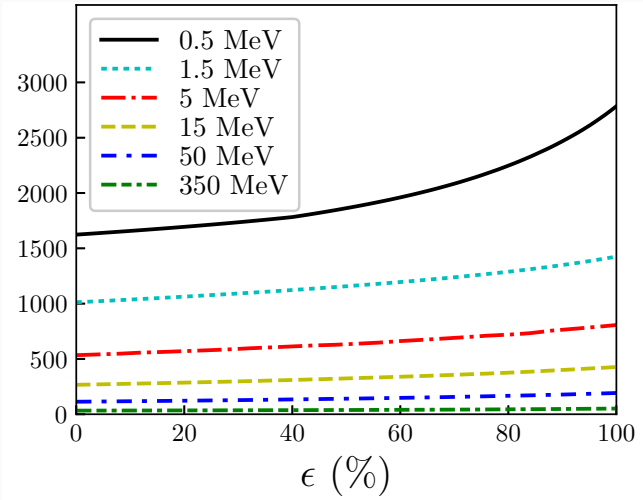
The midpoint rule
of the Newton-Cotes formula:

$$\Rightarrow \Delta E = s \cdot L_{eq}(E, \epsilon)$$
$$L_{eq}(E, x) = \frac{L(x)}{\left[1 + \frac{2L'(x)^2 - L(x)L''(x)}{L(x)^2}\,\frac{E^2 + x(x-2E)}{6}\right]}$$
$$x = E\left(1 - \tfrac{\epsilon}{2}\right) \text{ and } \epsilon(s, E) = \frac{\Delta E}{E}$$

ϵ links the ΔE space with the s space.

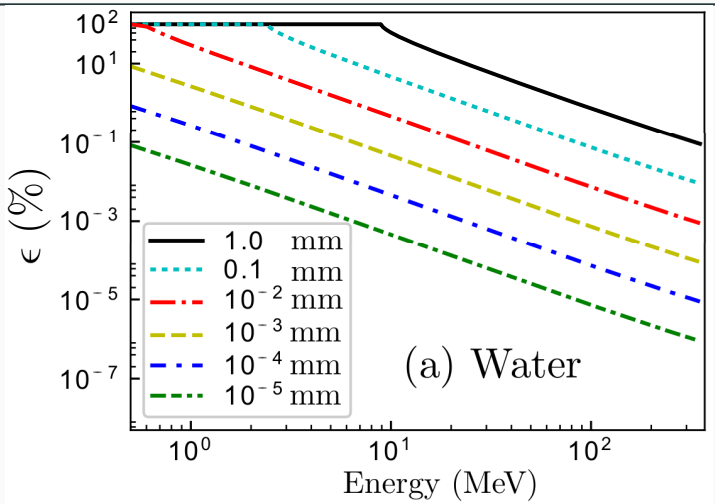
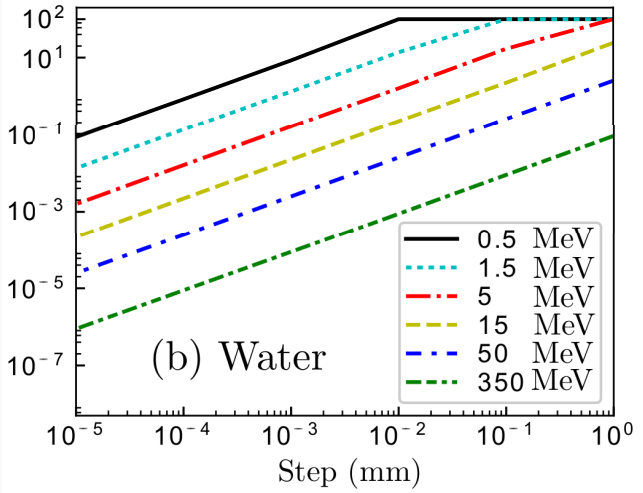
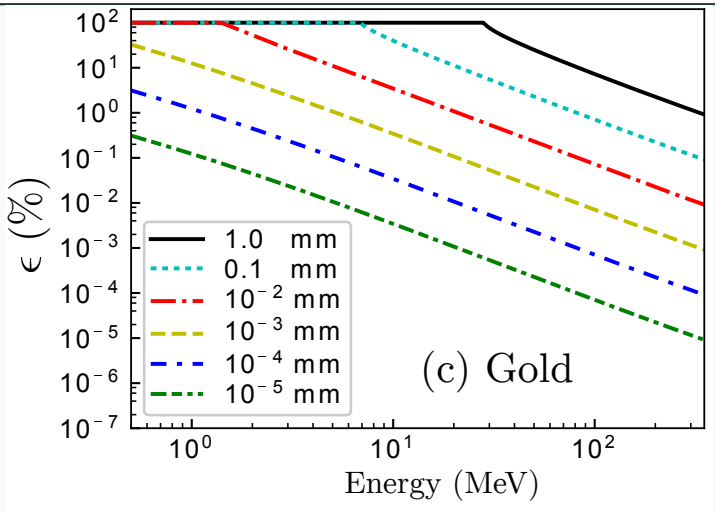
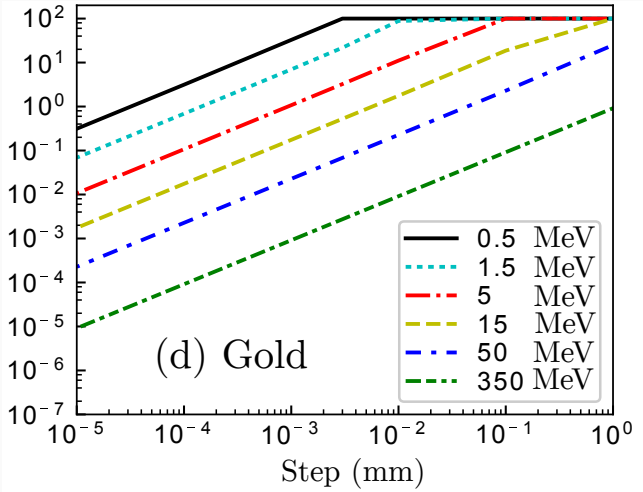
L_{eq} Look-up tables

$$L_{eq}(E,x) = \frac{L(x)}{\left[1+\frac{2L'(x)^2 - L(x)L''(x)}{L(x)^2}\frac{E^2+x(x-2E)}{6}\right]} \quad \text{with } x = E\left(1-\frac{\epsilon}{2}\right)$$

Material	Fixed ϵ	Fixed Energies
Water		
Gold		

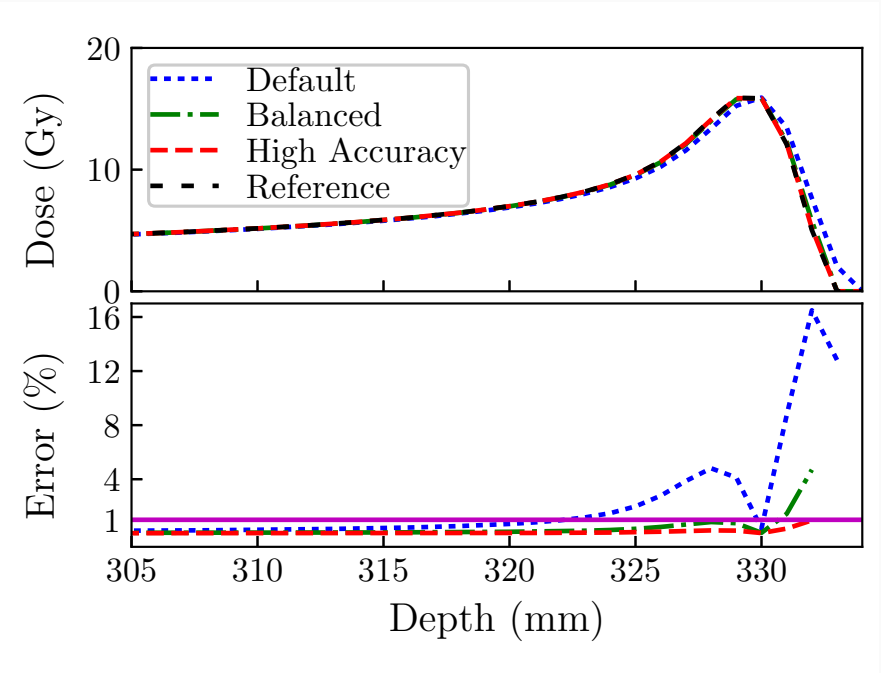
ϵ look-up tables

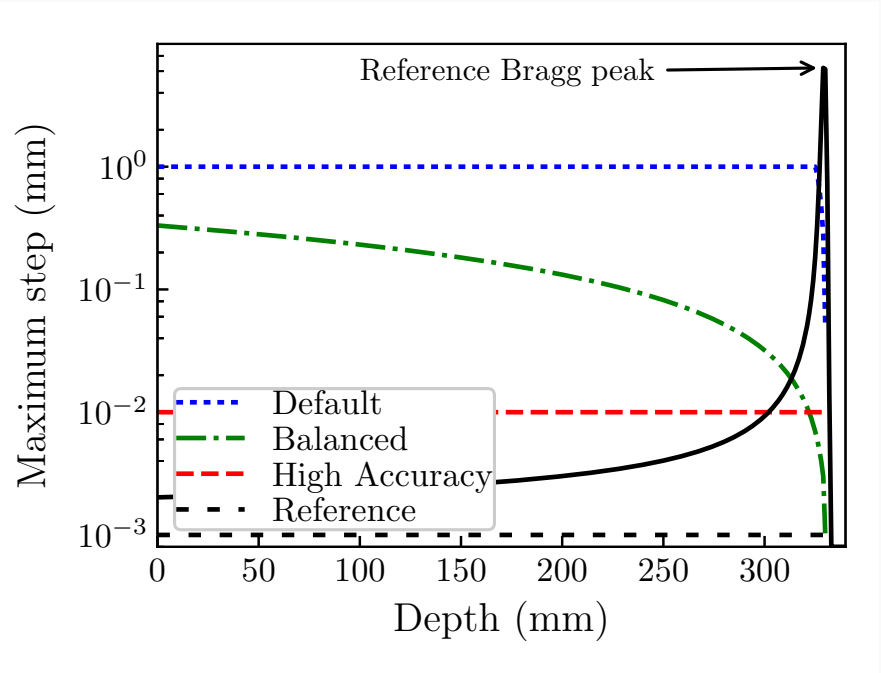
$\epsilon(s, E) = \frac{\Delta E}{E}$ determined when solving $s + \int_E^{E(1-\epsilon)} \frac{\mathrm{dE}'}{L(E')} = 0$

Material	Fixed Step lengths	Fixed Energies
Water		
Gold		

Geant4 setups

Simulation setups	Geant4 CSDA parameters			Time (h)	Errors (%)	
	d_{max}	Linear loss	`Step function`		Maximum	Falloff
Reference	1 µm	1%	(20%, 50 µm)	140	-	-
High Accuracy (HA)	10 µm	1%	(20%, 50 µm)	14	0.2	0.9
Balanced (Bal)	1 mm	0.1%	(0.1%, 1 µm)	2.5	0.8	4.7
Default	1 mm	1%	(20%, 50 µm)	0.8	4.8	16.5





GPU-based Implementations

Graphic processor Unit (GPU)

- pGPUMCD: a new GPUMCD branch dedicated to proton MC transport
- GPUMCD: a validated GPU-based MC dose calculation code for photons and electrons (Hissoiny et al., 2011c)

Efficiency

L_{eq} formalism intrinsic efficiency:

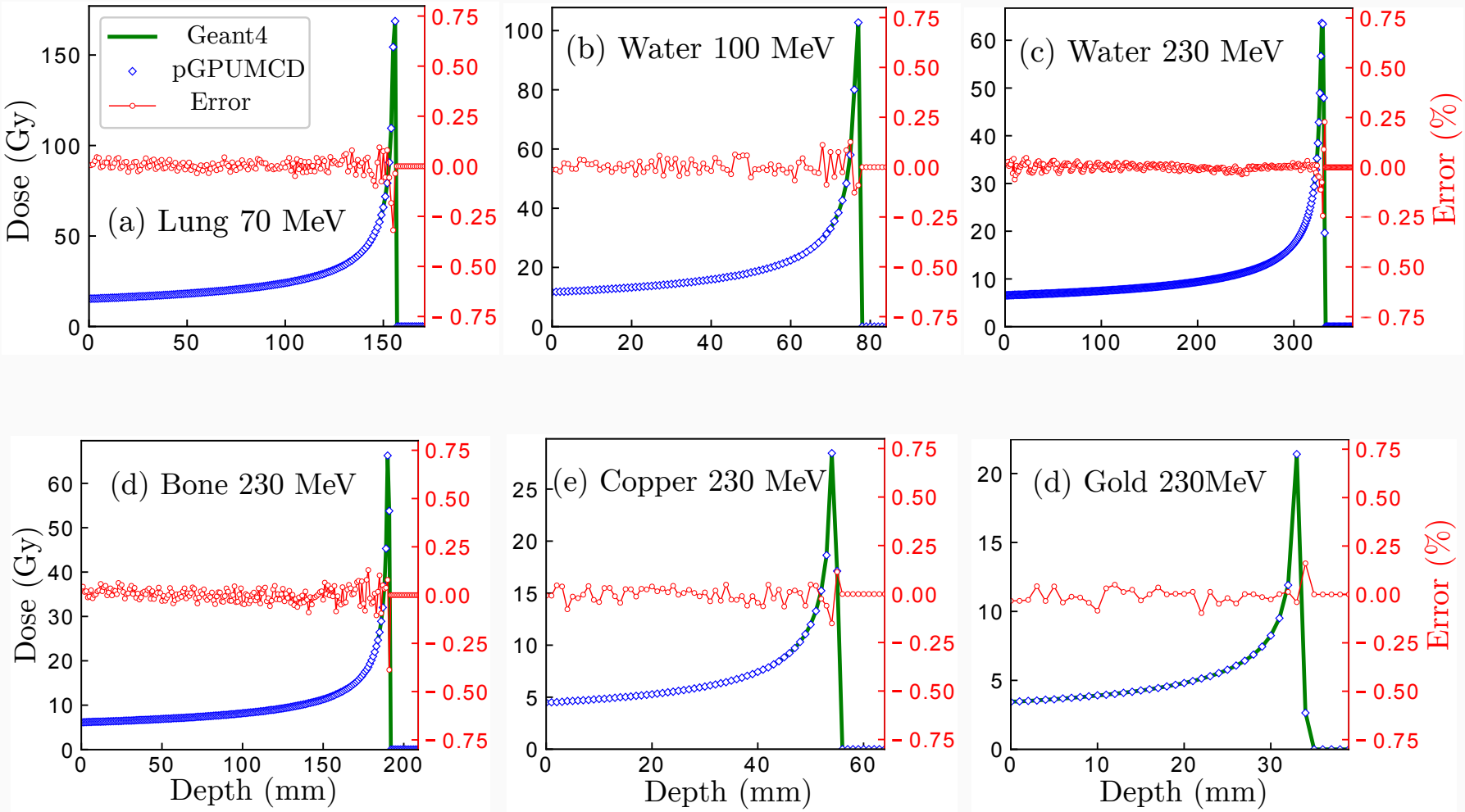
- pGPUMCD-L: the Geant4 CSDA scheme
- pGPUMCD-L_{eq}

Material	ρ	n_e $(\times 10^{23})$	I	T_e^{min}
	$\mathrm{g.cm^{-3}}$	$\mathrm{cm^{-3}}$	eV	MeV
Lung*	0.26	0.86189	69.69	0.148
Water	1.0	3.3428	78	0.352
Bone**	1.85	5.9056	91.9	0.512
Copper	8.96	24.625	322	1.4
Gold	19.32	46.665	790	2.3

* ICRU inflated lung (ICRU, 1992)

** Bone, Compact (ICRU) (Berger et al., 2005)

L_{eq} formalism validation



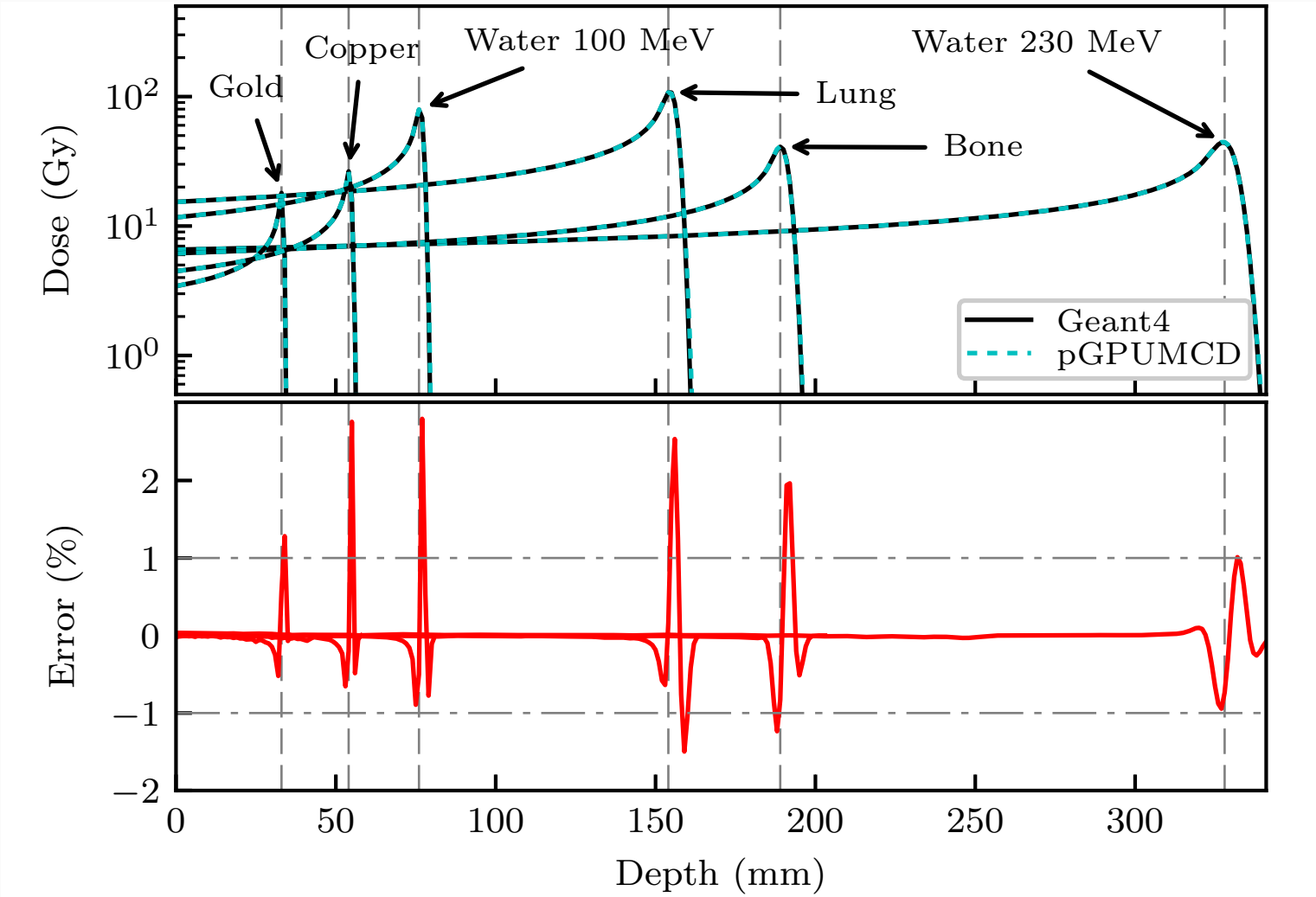
The ranges (R_{80}) matched within 1 μm

L_{eq} formalism efficiency

		Geant4		pGPUMCD-L		pGPUMCD-L_{eq}	Intrinsic speed up factors			
Material	Energy (MeV)	T_{G4}^{HA} (**hour**)	T_{G4}^{Bal} (**hour**)	T_{L}^{HA} (**second**)	T_{L}^{Bal} (**second**)	$T_{L_{eq}}$ (**millisecond**)	GPU (T_{G4}/T_{L}) HA	GPU (T_{G4}/T_{L}) Bal	L_{eq} ($T_{L}/T_{L_{eq}}$) HA	L_{eq} ($T_{L}/T_{L_{eq}}$) Bal
Lung	70	6.5	2	12.25	2.85	70	1,900	2,500	175	41
Water	100	3.5	1.9	4.73	2.12	46	2,600	3,200	103	46
Water	230	14	2.5	91	4.34	145	550	2,100	630	30
Bone	230	8	2.3	17.62	3.05	84	1,600	2,700	210	36
Copper	230	2.5	1.6	4.11	1.64	40	2100	3,500	103	41
Gold	230	1.7	1.4	3.23	1.30	31	1900	3,800	103	42

- Geant4/EGSnrc: linear algorithmic time complexity, *i.e.* $O(n)$ where n represents the number of subdivision in a voxel to maintain the mean energy loss accuracy. n is fixed by d_{max}.
- the L_{eq} formalism: constant algorithmic time complexity, *i.e.* $O(1)$

L_{eq} formalism with the energy straggling



The ranges (R_{80}) matched within 100 μm

Computation times

Geant4:

1.4 to 20 hours per million transported protons

`pGPUMCD:`

31 to 173 milliseconds per million transported protons

Conclusion

The L_{eq} formalism led to an intrinsic efficiency gain factor ranging between 30-630, **increasing with the prescribed accuracy** of simulations.

It allows **larger steps** leading to a **constant algorithmic time complexity**. It significantly accelerates Monte Carlo proton transport while **preserving accuracy**.

The L_{eq} formalism constitutes **a promising variance reduction technique** for computing proton dose distributions in a clinical context.

The L_{eq} formalism could be used for other charged particles.

The multiple scattering was validated, not presented here.

Under investigations: nuclear interactions

More details concerning the L_{eq} formalism: **(Maneval et al., 2017)**

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